

Final Exam , MTH 205, Fall 2014

Ayman Badawi

QUESTION 1. (10 points) Find the general solution to $\frac{dy}{dx} = \frac{1}{0.5x-(y+0.5y^2)x^3}$

QUESTION 2. (12 points, each = 4 points)

(i) find $\ell^{-1}\{\frac{5^{-s}}{(s-1)^2}\}$

(ii) Find $\ell^{-1}\{\frac{s+1}{(s-2)^2+9}\}$

(iii) Find $\ell\{\int_0^x e^{2x-5r} sin(r)\ dr\}$

QUESTION 3. (8 points) Solve for $y(x)$: $y^{(2)} + \frac{y'}{x+1} - \frac{y}{(x+1)^2} = \frac{10}{(x+1)^2}$. Given $y(x) = x+1$ is a solution to the associated homogenous equation. [Hint: For this particular equation, I should not give you one solution to the homogenous part!!!, but anyway I did]

QUESTION 4. (7 points) Solve for $y(x)$: $xy^{(2)} + 2y' + \frac{12.5y}{x} = 0$ (assume $x > 0$).

QUESTION 5. (10 points) Given that $f(x)$ is PERIODIC and defined on the interval $[0, \infty]$. The first period of $f(x)$ is determined by

$$\begin{cases} 1 & if \ \ 0 \leq x < 2 \\ 0 & if \ \ 2 \leq x < 4 \end{cases}$$

Solve for $y(x)$ if

$$\int_0^x f(r)\, dr + \int_0^x f(x-r)y'(r)\, dr = cos(x), y(0) = 1$$

[Hint: note that $1 - e^{-4s} = (1 - e^{-2s})(1 + e^{-2s})$ and $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$]

QUESTION 6. (12 points) Given $y' = -y^3 + 16y$.

a) Find the critical points of the D.E, and label each as STABLE, SEMI-STABLE, NON-STABLE.

b)If the graph of a solution to the D.E is passing through the point (0, 4), then sketch a rough graph of this solution.

c)Suppose if we have decided to assume that $y(0) = \frac{4}{3}$. Then solve for $y(x)$, i.e., solve the given D.E. [Hint: I do not recommend separation method here!!, maybe messy calculations if you do]

QUESTION 7. (7 points) Solve for $y(x)$: $y^{(2)} + 6y' + 9y = 4e^{-3x}$ [Hint: be wise when calculating y_p].

QUESTION 8. (8 points) Imagine that there is an object weighing 4 pounds stretches a spring $\frac{64}{25}$ feets. Assume that an air-resistance is numerically equals to $\frac{1}{8}$ of the velocity of the motion x(t) acts on the system. a) Determine the equation of motion x(t) if the object is initially released from 0.5 foot below the equilibrium position with an upward velocity $\frac{3}{2}ft/s$.

QUESTION 9. (8 points) Imagine that there is an electric source of an electric circuit given as $E(t) = 10(sin(t) + cos(t))$, the resistor-constance $R = 5$ Ohms, the capacitor-constant $c = 0.2$ Farad (No inductor is attached to the circuit). Initially, the charge on the capacitor is 2. Find the current $i(t)$ in the circuit at any time t. Find the steady-state-current.

QUESTION 10. (8 points)

Is $\frac{dy}{dx} = \frac{-(cos(x)y+2x-3)}{sin(x)+cos(y)-4y^3}$ exact? if yes, then solve it. If no, then please develop a new method that will help us to solve it.

QUESTION 11. (10 points) Let $A(t)$ be the amount of salt at any time t. Now imagine there is a tank initially holds 32 gallons of a mixture containing 12kg of salt. A mixture containing 1 kg of salt per gallon is poured into the tank at the rate 4 gal/min, while the well stirred mixture leaves the tank at rate 4 gal/min. a) When will the amount of salt in the tank be doubled? (i.e., find t so that the amount of salt is 24.). b) Find the concentration of salt per gallon after 32 minutes.

Faculty information

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